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D4.1: Optimized wave buoy layout

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EXECUTIVE SUMMARY

The CETO real-time control system requires the prediction of wave excitation forces over a 15-s time horizon, achieved using up-wave measurements from distant wave buoys. This report describes a methodology to optimize the number and locations of such wave buoys, considering the environmental conditions at the Biscay Marine Energy Platform (BiMEP) test site, where the future CETO pilot unit will be deployed.

The wave buoys measure the vertical and horizontal displacements of the sea surface. A novelty of this work is the consideration of the slow, current-induced variations of the buoy position, along the watch circle allowed by its mooring arrangement. The possible data transmission latency between the buoys and the CETO control system is also discussed and successfully included in the optimization. In addition to wave buoy data, (past) excitation force estimates, calculated in the CETO control system, can also be included among the input variables of the predictor. A 0.5 s measurement sampling time and prediction update period is found both sufficient and realistic.

The statistical method for real-time forecasting (SM-RTWF) serves as a fundamental building block in the optimization cost function calculation: With a given “candidate” measurement configuration (including the buoy layout, the choice of observed variables, as well as the predictor temporal settings), and in a given sea state, the SM-RTWF approach calculates the achievable excitation prediction performance, in terms of normalized mean-square error (NMSE), for each time horizon and degree of freedom. The cost function is calculated by combining those NMSE values across time horizons, degrees of freedom, sea states, and current-induced buoy position variations, using appropriate weights. The cost function computation can consider the wave buoy complementarity with radar-based wave measurements, by recalculating environmental statistics, and hence weights, restricted to those conditions where the radar is not reliable. However, it is uncertain at this stage whether X-band radar wave measurements will be available near the BiMEP pilot unit; therefore, the results presented in this report are for the full set of wave conditions at BiMEP.

The cost function calculation is further embedded within a relatively simple optimization algorithm, whereby wave buoys are added iteratively, and each new buoy is optimally placed using an exhaustive search approach. For each optimized configuration, numerical simulations are run and provided as inputs to a simulator of the CETO real-time control system, to estimate the resulting AEP.

Several modelling scenarios and optimization options are investigated. In particular, the benefit of including (or not) excitation estimates among observed variables, and that of weighting (or not) the prediction horizon to give preference to shorter time horizons, are assessed. The sensitivity of the optimal layout and resulting performance to the buoy mooring watch circle is also investigated, as well as the impact of data transmission latency time.

Depending on the modelling scenario considered, between 1 and 5 optimally placed buoys are sufficient to allow AEP losses below 0.5%, with respect to that, obtained with perfect predictions, thus demonstrating the efficacy of the proposed optimization methodology. However, results also show that the optimal layout, which allows such performance, strongly depends on the modelling scenario, which stresses the importance of duly considering those modelling parameters, when optimizing the buoy layout.

In terms of compromise between feasibility and performance, the retained reference scenario is that of a wave predictor using wave buoy measurements only (i.e. not including excitation estimates among the observed variables), and a mooring arrangement allowing a 15m-radius buoy watch circle. In such a case, no appreciable AEP improvement is observed beyond $N = 4$ buoys, and the AEP value is hardly different from the perfect-prediction case.

A precise characterization of the data transmission latency time would require field testing of the future real-time communications architecture in the CETO farm. Therefore, in this report, only a sensitivity analysis is shown, assuming a (likely exaggerated) 5s latency time. A rather substantial impact onto buoy layout and AEP is highlighted, requiring 1 additional buoy (i.e. 5 buoys in total) to reach performance levels similar to the no-latency case.

Finally, if it is found that excitation estimates can be included among the observations with great profit, requiring even fewer buoys to achieve the same performance. However, it should first be ascertained whether those estimates can be used in a real-time setup, without inducing unwanted closed-loop instability in the overall estimation-prediction-control process.

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1 INTRODUCTION

Optimal control of wave energy converters (WECs) requires the prediction of future wave excitation forces, over time horizons in the order of a few seconds [1]. Such predictions are referred to as real-time wave forecasting (RTWF). For very short time horizons (smaller than a wave period), RTWF may be approximately achieved using only sensors on-board the WEC, but longer horizon requirements call for up-wave measurements, whereby the free-surface elevation (FSE) is measured in one or more locations up-wave with respect to the WEC, before the measured information is propagated in time and space to the WEC location [2].

RTWF is one of the challenges addressed by the COIN project, within Work Package 4. More specifically, CETO's advanced controller requires predictions of future wave excitation forces onto the device. Up-wave measurements may be achieved by means of "traditional" local sensors such as wave buoys, or by means of remote sensing instruments such as lidars, stereo cameras or X-band radars. The COIN project considers a farm-level measurement and monitoring system, comprised of a farm-level, X-band radar wave measurement system (with large range, but limited resolution and availability), and arrays of wave buoys that measure the wave elevation locally, as illustrated in Figure 1.

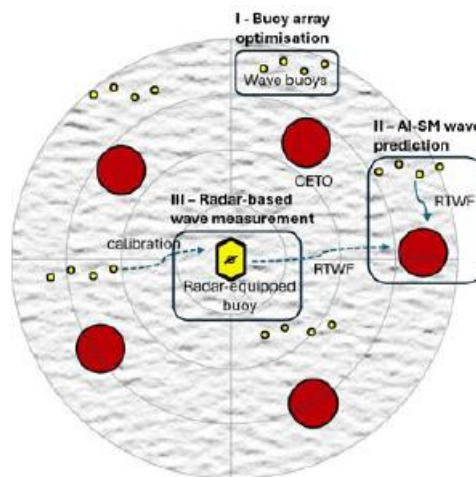


Figure 1: Farm-level wave monitoring and real-time prediction architecture.

The specific task T4.1 within Work Package 4 is concerned with the optimization of such an array of wave buoys, for an individual CETO device. More precisely, a methodology is outlined and implemented, to optimize the layout of an array of wave measurement buoys, to maximize the CETO control performance, taking into account the prevailing wave conditions at the Biscay Marine Energy Platform ([BiMEP](#)) deployment site.

In the prospect of deploying those wave buoys together within a farm-level measurement and monitoring system, it is important that the proposed optimization methodology can take profit from the complementarity between the wave buoys and radar. Therefore, it will be outlined how it can be ensured that the wave buoy layout is optimal, primarily in those sea states where the radar is less likely to be operational (mean wind speeds below 3 m/s, and significant wave heights below 1.5 m). However, substantial uncertainty is still associated with radar-based wave measurements¹, which makes it difficult at this stage to know precisely in which conditions the radar will provide reliable measurements. Furthermore, in the first CETO deployment at the BiMEP site, radar availability has not

¹ uncertainty which the COIN project will address at a later stage.

been confirmed. Therefore, the optimization methodology will be adaptable to ensure radar complementarity, but the results presented here will be for the whole set of CETO's operational conditions, conservatively assuming that no radar system will be available.

Attention must be given to real-time sampling and communication aspects between the wave buoys and the CETO. Indeed, for RTWF, the wave buoy array needs to be able to transmit wave displacement measurements in near real-time, with a high sampling rate and short latency, compatible with control requirements. It will be shown how those aspects are considered in the buoy layout optimization.

During the implementation of Task 4.1, an additional challenge emerged, that had not been anticipated: While the anchoring point of each wave buoy can be decided accurately, it was realised that the buoys itself could undergo relatively large variations in their horizontal positions, depending on their mooring system and prevailing conditions. More specifically, for a typical Spotter buoy² deployed at the BiMEP site with its recommended mooring arrangement, the buoy watch circle exceeds 100 m in diameter. Such an effect thus had to be considered in the optimization. Furthermore, the impact of the buoy watch circle radius onto the mean prediction performance, and therefore, onto WEC mean annual power capture, could be assessed, thus providing guidelines for the choice of appropriate mooring designs for the wave buoy array.

The rest of this deliverable is organized as follows:

- Section 2 describes the stochastic approach to wave prediction (SM-RTWF), which serves as the fundamental building block of the optimization process;
- Section 3 briefly introduces the CETO device with its control system, and expresses its wave prediction requirements;
- Section 4 focuses on the BiMEP test site, where the next deployment of a CETO prototype will take place, and which is the reference wave energy site for WP4. In particular, it is explained how representative sea states are selected in view of the buoy layout optimization.
- In Section 5, the practicalities associated with wave measurement buoys are discussed: What buoy technology and size would be suitable? What are possible sampling rates in comparison to WEC control requirements? How can real-time data transmission be achieved, and how are those constraints taken into account in the predictor and in the SM-RTWF predictor and in the buoy layout optimization? What are typical buoy watch circles and how do should they be modelled in the optimization?
- Section 6 summarizes the layout optimization methodology.
- Section 7 presents the optimization results.
- Conclusions are formulated in Section 8.

² <https://www.sofaroccean.com/products/spotter>

2 STOCHASTIC FRAMEWORK FOR WAVE PREDICTION AND BUOY LAYOUT OPTIMIZATION

The stochastic method for RTWF (SM-RTWF) [3] [4] relies on a rigorous mathematical and statistical framework, which, given known sea conditions (i.e. a known wave spectrum), and given a measurement configuration (i.e. a set of measured variables at a number of locations, during a given time interval), calculates the statistically-optimal predictor in a mean-square error sense, together with the prediction mean square error for each time horizon. Therefore, the approach is a natural way to calculate, in each sea state, the prediction accuracy that can be obtained from a given wave buoy layout, thus providing the fundamental building block of the buoy layout optimisation.

2.1 Assumptions

The SM-RTWF relies on assumptions that have been commonplace in oceanography and marine engineering since the 1950s [5]: On the short time and space scales of RTWF (less than a minute, less than a kilometre), the wave field is modelled as a zero-mean, stationary, homogeneous and Gaussian random field. The statistical properties of this random field are entirely encapsulated by the frequency-direction spectrum $S_\eta(\omega, \theta)$ of the free-surface elevation η , where ω is the angular frequency and θ the propagation direction.

Under such assumptions, the wave prediction problem can be formulated in a rigorous statistical framework, as shown by [4] and [3]. Like in [6], we extend the formulation of those two references to account for variables different from the free-surface elevation.

We start by considering a pair of variables, v_i and v_j , linearly related to the FSE η through transfer functions $H_i(\omega, \theta)$ and $H_j(\omega, \theta)$, and taken at two horizontal locations, $\vec{x}_i, \vec{x}_j$ and two instants $t + \tau_i$ and $t + \tau_j$, where t is the present time. Then, because of the Gaussian, stationary and homogeneous assumptions, v_i and v_j jointly follow a zero-mean Gaussian distribution, with covariance $\langle v_i(t + \tau_i)v_j(t + \tau_j) \rangle$ that can be calculated using the Wiener-Khinchine relation [7], applied to the cross-spectrum S_{ij} of variables v_i and v_j :

$$\langle v_i(t + \tau_i)v_j(t + \tau_j) \rangle = \frac{1}{2\pi} \int_0^\infty \int_0^{2\pi} S_{ij}(\omega, \theta) e^{j[\vec{k}(\omega, \theta) \cdot (\vec{x}_j - \vec{x}_i) - \omega(\tau_j - \tau_i)]} d\theta d\omega \quad (1)$$

where the cross-spectrum S_{ij} is given by:

$$S_{ij}(\omega, \theta) = S_\eta(\omega, \theta) H_i^*(\omega, \theta) H_j(\omega, \theta) \quad (2)$$

and $\vec{k}(\omega, \theta)$ is the wave vector associated with direction θ and frequency ω .

2.2 Prediction problem formulation

Let Δt be the sampling time of the measurement and prediction systems. A number M_o of variables v_i , $i = 1 \dots M_o$, are observed and used as inputs to the predictor. Each variable is measured at a given horizontal position $\vec{x}_i$, and is related to η through its directional transfer function $H_i(\omega, \theta)$. The aim of the predictor is to forecast a number M_p of variables w_j , $j = 1 \dots M_p$, each of them with a given horizontal position $\vec{x}_j$, and related to η through its directional transfer function $H_j(\omega, \theta)$. The prediction is implemented in a receding horizon fashion, as required, for example, by a WEC receding-horizon controller [1]. More specifically, let t denote the present time, i.e. the instant at which the prediction calculations are performed. The predicted variables need to be predicted at N_p time instants $t + \tau_{p,min}$, $t + \tau_{p,min} + \Delta t, \dots, t + \tau_{p,max}$ where $N_p = \frac{\tau_{p,max} - \tau_{p,min}}{\Delta t} + 1$, using the observed variables at N_o time instants $t + \tau_{o,min}$, $t + \tau_{o,min} + \Delta t, \dots, t + \tau_{o,max}$ where $N_o = \frac{\tau_{o,max} - \tau_{o,min}}{\Delta t} + 1$, with the constraint that $\tau_{o,max} \leq 0$ since observed variables can only be observed up to present time.

$\forall t \in \mathbb{R}$, define the following vectors:

$V(t) = \begin{bmatrix} v_1(t) \\ \vdots \\ v_{M_o}(t) \end{bmatrix}$, $W(t) = \begin{bmatrix} w_1(t) \\ \vdots \\ w_{M_p}(t) \end{bmatrix}$ which concatenate observed and predicted variables, respectively, at a given instant t .

Now define the vector which concatenates all the observations, taken into account by the predictor:

$V(t) = \begin{bmatrix} V(t + \tau_{o,min}) \\ \vdots \\ V(t + \tau_{o,max}) \end{bmatrix}$ and, similarly, the vector which concatenates all the values, that the predictor must predict: $W(t) = \begin{bmatrix} W(t + \tau_{p,min}) \\ \vdots \\ W(t + \tau_{p,max}) \end{bmatrix}$.

The prediction problem is to find a predictor P , such that $\widetilde{W}(t) = P(V(t))$ minimizes some error metrics with respect to $W(t)$.

For example, assuming two wave buoys at positions $\vec{X}$ and $\vec{X}'$, each of them measuring 3 variables (typically, the vertical and the two horizontal free surface displacements), the number of observed variables v_i is $M_o = 3 \times 2 = 6$, and their locations are $\vec{x}_i = \vec{X}$ for $i = 1 \dots 3$ and $\vec{x}_i = \vec{X}'$ for $i = 4 \dots 6$. The predicted variables w_j , in this work, are the WEC excitation forces in 5 degrees of freedom, the WEC being at a location $\vec{X}''$, so that $M_p = 5$ and $\vec{x}_j = \vec{X}''$, $j = 1 \dots 5$. The practical application of the method to the CETO excitation force prediction problem is further detailed in Section 6.

2.3 Prediction problem solution

Using the Gaussian assumptions, $V(t)$ and $W(t)$ jointly form multidimensional, zero-mean Gaussian random vectors, with covariance matrices $\Sigma_{oo} = \langle V(t)V^T(t) \rangle$, $\Sigma_{pp} = \langle W(t)W^T(t) \rangle$ and $\Sigma_{op} = \langle V(t)W^T(t) \rangle = \Sigma_{po}^T$. Furthermore, using the stationarity and homogeneity assumptions, those

covariance matrices are time-independent, and each of their entries can be filled-in using the general relation of Equation (1).

The prediction $\widetilde{\mathbf{W}}(t)$, which minimises the mean-square error with respect to $\mathbf{W}(t)$, knowing the observations $\mathbf{V}(t)$, coincides with the conditional expected value of $\mathbf{W}(t)$ knowing $\mathbf{V}(t)$, and is calculated as follows:

$$\widetilde{\mathbf{W}}(t) = \mu_{\mathbf{W}(t)|\mathbf{V}(t)} = \mathbf{P} \mathbf{V}(t) \quad (3)$$

where the prediction matrix $\mathbf{P}$ is time-independent, calculated as:

$$\mathbf{P} = \boldsymbol{\Sigma}_{po} \boldsymbol{\Sigma}_{oo}^{\dagger} \quad (4)$$

and the $\dagger$ (dagger) superscript denotes the inverse – if $\boldsymbol{\Sigma}_{oo}$ is full-rank – or a pseudo-inverse – if $\boldsymbol{\Sigma}_{oo}$ is not full-rank, which indicates that the information in $\mathbf{V}(t)$ is statistically redundant, as shown in [3].

The corresponding prediction error $\mathbf{E}(t) = \widetilde{\mathbf{W}}(t) - \mathbf{W}(t)$ is also stationary, zero-mean, Gaussian, with covariance matrix $\boldsymbol{\Sigma}_{p|o}$ calculated as follows:

$$\boldsymbol{\Sigma}_{p|o} = \boldsymbol{\Sigma}_{pp} - \mathbf{P} \boldsymbol{\Sigma}_{op} \quad (5)$$

In particular, the diagonal terms of $\boldsymbol{\Sigma}_{p|o}$ provide the achievable mean-square prediction error for each predicted variable w_j and time horizon τ . After appropriate normalisation by the variance of w_j , we obtain the achievable normalised mean square error profile, $NMSE_j(\tau)$, for predicted variable j and time horizons $\tau = \tau_{p,min}, \tau_{p,min} + \Delta t, \dots, \tau_{p,max}$.

2.4 Limitations

The SM-RTWF approach is currently limited to Gaussian, random wave fields, as discussed in [3]. Therefore, its performance is expected to deviate from the theory in strongly non-linear wave fields, where wave statistics significantly depart from Gaussianity. Those “non-linearity limitations” will be addressed in future stages of Work Package 4.

Furthermore, the method is limited to *random* wave fields, implying that specific events, such as the waves of a ship’s wake, may not be predicted as accurately as with a purely physics-based numerical model.

Finally, the SM-RTWF approach, as described in this section, requires knowledge of the wave spectrum. As a consequence, it is sensitive to possible errors in the online estimation of wave conditions, in terms of frequency and directional distribution.

Nevertheless, none of those limitations are expected to play a significant role in the choice of an optimal buoy layout, which allows us to safely use the SM-RTWF approach as the main tool for this task.

3 PREDICTION REQUIREMENTS FOR THE CETO DEVICE

3.1 CETO device

The CETO system is a fully submerged point-absorber wave energy converter developed by Carnegie Clean Energy. It uses a shallowly submerged cylindrical Buoyant Actuator (BA) that moves with wave-induced orbital motion and drives a power-take-off (PTO) system to produce electricity. The BA is taught-moored with three legs, each connected to a separate seabed foundation at the seabed, and wrapped around a drum at the other end, within its dedicated PTO module. The electric PTOs control the tether load and convert the BA motions into reciprocating rotary motions before converting it into electrical power, enabling energy capture from multiple degrees of freedom (DoFs). The system used as the basis for this study has a 20 m diameter by 4 m tall BA, moored in 70 m of water.

3.2 Advanced MPC controller

CETO uses an advanced control algorithm to maximise energy extracted from the waves. One such controller belongs to the family of Model Predictive Control (MPC) algorithms, and was used as the reference controller for this study. At each timestep, the MPC requires a prediction of the wave excitation forces over a prediction horizon T_h . With this input, it uses an internal model of CETO to run an online optimisation aiming to maximise electrical power over that prediction horizon. The output of the controller is the optimum PTO forces timeseries over T_h , out of which only the first timestep is applied (in a receding horizon fashion), as the controller moves to the next timestep

At CTS, the MPC controller is embedded within a WEC-Sim³ numerical simulation model of the CETO, based on a Matlab-Simulink⁴ environment. This simulator, hereafter referred to as the MPC simulator, is employed in several ways in the scope of Task 4.1:

- To determine the MPC prediction requirements in terms of time horizon, time step and accuracy;
- To calculate the CETO power matrix, $P^*(H_s, T_p)$, assuming perfect excitation prediction – this is useful to assign a weight to each of the sea states considered in the buoy layout optimization, as described in Section 4;
- To allow the accurate evaluation of the mean annual power capture, allowed by the wave buoy layouts, optimized according to different optimization scenarios (see Section 7)

3.3 Excitation force estimation

Wave excitation forces onto the WEC are not directly measurable quantities, which is why WEC control systems often feature a wave excitation force estimator, based on measured WEC dynamics and control forces [8]. Such an estimator was successfully implemented in [9] for a CETO-like device. Therefore, it can be considered that excitation forces are estimated up to present time t . Since the SM-RTWF method can combine measurements of different types, as shown in Section 2, those excitation

³ <https://wec-sim.github.io/WEC-Sim/main/>

⁴ <https://fr.mathworks.com/products/matlab.html>

estimates can be included among the observed variables with great profit. This scenario will also be investigated in the optimization, and the results will be compared with those obtained when no excitation estimator is available, thus allowing an assessment of the benefits that can be expected from developing an excitation force estimator for the CETO device. CTS is planning to implement a wave excitation estimator in the control system of future deployed units.

3.4 Prediction requirements

Ideally, the prediction requirements should be specifically those of the Health-aware controller (HAC), which will be developed during the COIN project. However, since the HAC is not yet developed, it is reasonable to work with the prediction requirements of CETO's current MPC control system.

3.4.1 Predicted variables and associated weights

The CETO exploits the wave motion in 5 degrees of freedom: surge, sway, heave, pitch and roll, while the yaw motion has no effect onto power absorption. Therefore, the MPC inputs are the future excitation signals, $e_{j,j=1\dots 5}(t + \tau)$, in those 5 DoFs.

However, in a given sea state, not all 5 DoFs have the same relative contributions in the overall power. For instance, the motion aligned with the main wave direction tends to account for most of the absorber power, while the motion in the transverse direction has a minor contribution. In addition to the sea state, the relative DoF contributions also depend on the specific WEC kinematics. For those reasons, it would be sub-optimal to try to minimize the prediction error identically in all five DoFs. Instead, the cost function should take account of the relative weight of each DoF in terms of power contribution.

Therefore, in each sea state considered in the optimization (see Section 4), it is important to assign a weighting parameter to each DoF, representing its relative power absorption contribution in that specific sea state. For example, consider a sea state with an eastward mean direction (aligned with the CETO's x axis). In such conditions, most of the power absorption takes place in the heave, pitch, and surge degrees of freedom, while sway and roll undergo little excitation. If that information is not given to the optimizer, the latter will tend to also place wave buoys in the transverse direction (i.e. along the y axis) to predict sway and roll as accurately as the other DoFs, since sway and roll are excited by wave components propagating along the north-south axis. Those "transverse" buoys would incur additional costs, while having little or no benefits in terms of power absorption.

To determine a suitable weight, $\mathcal{W}_j^{DoF}$, for DoF j , in a given sea state n , idealised spectral control calculations are run, in the same way as [10], but without motion constraints or PTO efficiency considerations, for simplicity. It is also noted that surge and pitch are completely correlated degrees of freedom, and sway and roll too. Thus, surge-pitch, roll-sway, and heave, form three statistically independent excitation modes. The optimal power P^* is first calculated, using the spectral method, keeping all components of the excitation forces. Subsequently, by setting all excitation components but surge-pitch to zero, the optimal power P_{1-5}^* is calculated, which represents the surge-pitch power contribution. Similarly, the sway-roll contribution P_{2-4}^* and the heave contribution P_3^* are calculated. It can be verified that the total optimal power is recovered in the sum $P_{1-5}^* + P_{2-4}^* + P_3^* = P^*$. Finally, the following weights are assigned:

D4.1: Optimized wave buoy layout

- For surge and pitch: $\mathcal{W}_{j=1,5}^{DoF} = \frac{P_{1-5}^*}{P^* + P_{1-5}^* + P_{2-4}^*}$
- For roll and sway: $\mathcal{W}_{j=2,4}^{DoF} = \frac{P_{2-4}^*}{P^* + P_{1-5}^* + P_{2-4}^*}$
- For heave: $\mathcal{W}_{j=3}^{DoF} = \frac{P_3^*}{P^* + P_{1-5}^* + P_{2-4}^*}$

where the normalization in the denominator ensures that the sum of weights over all five DoFs is 1. Doing so in each of the optimization sea states n defined in Section 4, yields a sea-state set of weights $\mathcal{W}_j^{DoF}(n)$.

In a typical directional sea state of the BiMEP test site, as will be described in Section 4, approximately 42% of the power potential can be attributed to the DoFs aligned with the mean wave direction (i.e. surge and pitch, for a westerly mean wave direction), another 53% corresponds to heave, leaving 5% of the extracted power that can be attributed to the modes of motion, transverse to the mean wave direction (i.e. sway and roll, for a westerly mean wave direction).

3.4.2 Time step and prediction horizon

The optimization problem, solved by the controller at each time step, takes as an input the future excitation forces in the next T_h seconds, i.e. over the time interval $[t; t + T_h]$, with a time resolution Δt (the same as in Section 2). We can also define Δt_{update} , the update time of the excitation prediction. Thus, at time t the controller receives $e_{i=1\dots 5}(t, t + \Delta t, t + 2\Delta t, \dots, t + T_h)$. Then, at time $t + \Delta t_{update}$, the controller receives $e_{i=1\dots 5}(t + \Delta t_{update}, t + \Delta t_{update} + \Delta t, t + \Delta t_{update} + 2\Delta t, \dots, t + \Delta t_{update} + T_h)$, etc. For simplicity, we assume the time resolution and update time to be identical, i.e. $\Delta t_{update} = \Delta t$.

A sensitivity study carried out by CTS shows that $\Delta t = 0.5s$ ensures a favourable trade-off between computation time and power performance. The CETO controller's timestep is shorter than Δt , and required intermediate values are obtained using interpolation.

The value of T_h must be chosen large enough to ensure near-optimal control performance, while remaining computationally tractable. To that end, MPC simulations are run in typical sea states at the BiMEP deployment site (see Section 4), assuming perfect predictions. Simulations are performed, increasing the control horizon T_h gradually at each simulation, until the resulting power increase is negligible. We call T_h^* the optimized control horizon. Figure 2 shows an example of this procedure.

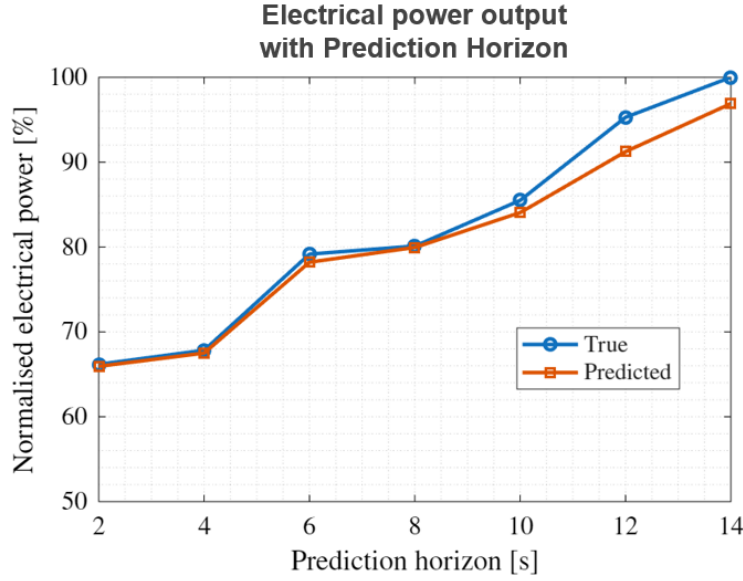


Figure 2: Increasing average electrical power output with prediction horizon in $H_s = 1$ m, $T_p = 10$ s. Blue: Ideal prediction, Red: Realistic prediction

3.4.3 Prediction accuracy requirements

It is straightforward that the controller's performance depends on the accuracy of the excitation prediction, provided by the predictor to the MPC. For the sake of simplicity, it would have been ideal to express prediction accuracy requirements in terms of a maximum average normalised mean-square error (NMSE) threshold over the time horizon T_h , as was initially considered. In that spirit, [11] suggests that a 13% NMSE threshold is necessary to achieve 95% of the optimal power performance⁵.

However, such a threshold assumes a homogeneous error profile over the prediction horizon, which is unrealistic. In practice, predictions tend to be better for short time horizons, and worse for longer horizons. Fortunately, the controllers are generally more sensitive to short time horizons. For example, [12] shows that near-optimal WEC trajectory can be calculated, while using a simple predictor with accuracy degrading rapidly with the time horizon. Thus, keeping a constant NMSE criterion over the prediction horizon poses the double risk of being insufficiently demanding for short time horizons, while setting unachievable and useless standards for long time horizons.

A horizon-weighting function is thus introduced, which penalises short-term prediction errors more strongly: $\mathcal{W}^{hor}(\tau) = \frac{(1+\tau)^{-\alpha}}{N}$, where α is a positive parameter, and N is a normalisation factor ensuring that the sum of $\mathcal{W}^{hor}(\tau)$, over the prediction horizon $[0; T_h^*]$, is 1. With $\alpha = 0$, equal weights are given to all time horizons, while with $\alpha = 1$, for example, preference is given to present and short time horizons.

⁵ Note that this reference mentions a 9% normalised root-mean-square error (NRMSE) criterion, where the NRMSE is normalised by H_s . In our conventions, the NRMSE would be normalised by the standard deviation of the free surface elevation, σ_η . Since $H_s = 4.01\sigma_{eta}$, a 9% NRMSE criterion in Sergiienko's conventions amounts to a 36% NRMSE in our conventions, i.e. a 13% NMSE.

4 SEA CONDITION MODELLING

4.1 The BiMEP deployment site

The Biscay Marine Energy Platform (BiMEP) is an open-water facility to support research, testing and demonstration of pre-commercial prototypes of marine renewable energy devices. It is located in the Basque Country (Spain), near the city of Bilbao, and covers a total surface area of 5.2 km² (see Figure 3). It includes four berths (numbered 1 to 4 from South to North), and CTS was awarded access to Berth #2, where the water depth is 70 m.

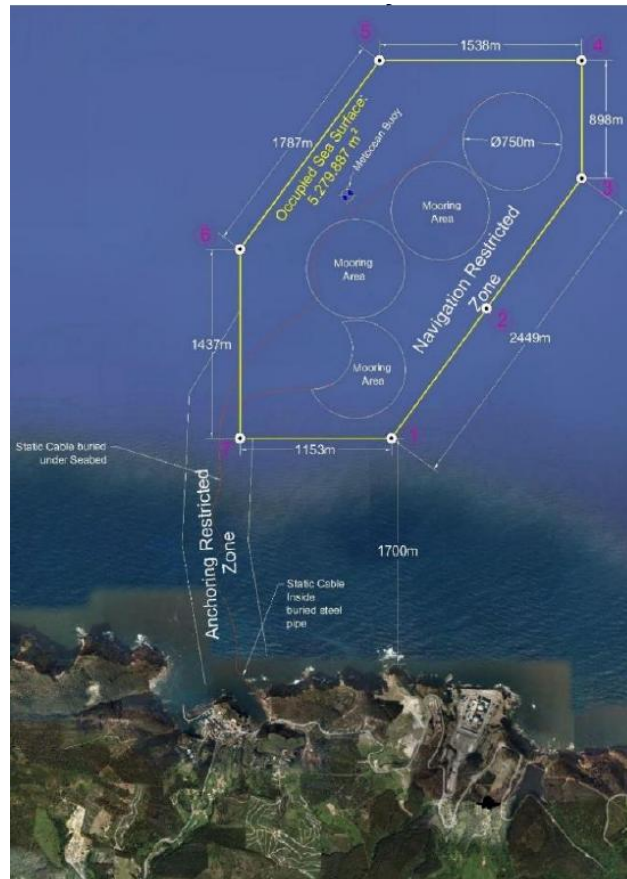


Figure 3: Aerial view and main dimensions of the BiMEP test site [13]

The prevailing wave conditions typically correspond to mild swell, with a most frequent sea state $H_s = 1$ m and $T_p = 10$ s, propagating from $\approx 310^\circ$ (with respect to geographical North), with a mean wave spreading of 22° . The site wave rose is shown in Figure 4.

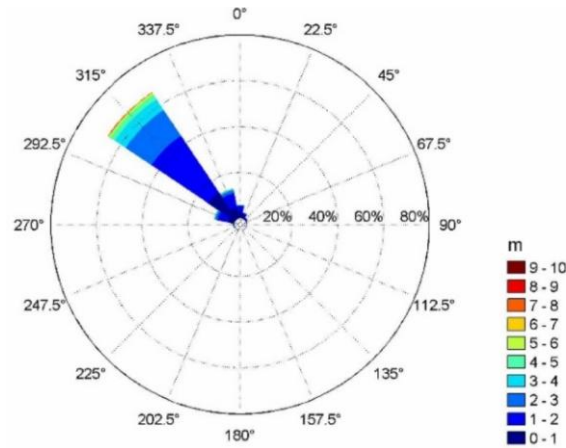


Figure 4: Hs Wave Rose of BiMEP conditions [14]

The following statistical analysis, to determine the buoy layout optimization sea states, is based on field data, obtained by BiMEP’s Wavescan wave buoy system (more details in BiMEP’s TRL+ report [14]), located at the northern boundary of the Navigation Restricted Zone, between February 2022 and December 2025.

4.2 Scatter diagram reduction for buoy layout optimisation

The buoy layout optimization needs to consider the prevailing wave conditions at CETO’s deployment location throughout the year. More specifically, the yearly average of the prediction NMSE⁶ is minimized in the optimization, which implies that at each cost function evaluation (corresponding to a buoy layout being evaluated), the predictor calculations of Section 2 are carried out in a potentially large number of sea states, representative of the BiMEP test site, thus incurring large computational costs. It is thus desirable to reduce as much as possible the number of representative sea states, where calculations will be carried out.

In principle, we thus need to:

- Assume a parametric model for the wave directional spectrum. The number of parameters determines the number of dimensions in the BiMEP site scatter diagram.
- Fill-in each entry of the scatter diagram using available BiMEP data.
- At each cost function evaluation, the predictor calculations of Section 2 are carried out in each entry of the scatter diagram.

We describe each sea states by means of a JONSWAP [15] frequency distribution, with a representative peak enhancement factor $\gamma = 3.3$. The directional distribution is modelled by a Mitsuyasu directional spreading function [16]. The wave spreading parameter is fixed to $s_{max} = 25$, which corresponds to a spreading angle of 23° , consistent with the mean wave spreading angle at the BiMEP site. There are

⁶ More detail about the cost function is provided in Section 6.

thus up to 3 remaining spectral parameters which describe the variety of BiMEP sea states, yielding a 3-dimensional scatter matrix; those are:

- significant wave height H_s ,
- peak wave period T_p ,
- mean wave direction $\bar{\theta}$.

Binning T_p by 2s from 3 to 21s, H_s by 1m from 0 to 4 m (CETO's upper operational limit) and $\bar{\theta}$ by 20° from 0 to 360° , a 3-dimensional scatter diagram $p(H_{s,k}, \bar{\theta}_i, T_{p,j})$ is filled-in, using the WaveScan wave dataset and bin-counting, and where the value p represents the annual probability (or frequency of occurrence) of each sea state bin. This diagram represents 648 sea states where prediction calculations would be carried out, for each cost function evaluation – an unnecessarily large number, incompatible with buoy layout optimization in a reasonable amount of time.

In order to reduce the number of sea states, it is first noticed that the NMSE results, as calculated in Section 2, are scale-independent, meaning that, for fixed values of the other spectrum parameters (T_p and $\bar{\theta}$), the NMSE only needs to be calculated for one, arbitrary value of H_s . Thus, the scatter diagram, which the cost function calculation needs to explore, is reduced to the two dimensions $\bar{\theta}, T_p$.

Next, the $(\bar{\theta}, T_p)$ pairs are ranked according to their contribution to the annual energy production (AEP), calculated as follows:

$$AEP(\bar{\theta}_i, T_{p,j}) = \sum_{k=1}^4 p(H_{s,k}, \bar{\theta}_i, T_{p,j}) P^*(H_{s,k}, T_{p,j}) \quad (6)$$

where $p(H_{s,k}, \bar{\theta}_i, T_{p,j})$ is the probability of bin k, i, j of the 3D scatter diagram, and $P^*(H_{s,k}, T_{p,j})$ is the entry k, j of the CETO power matrix. Finally, the first N_{sst} pairs $(\bar{\theta}, T_p)_n$, which together represent 90% of the AEP, are selected: As little as $N_{sst} = 12$ combinations $(\bar{\theta}, T_p)$ are found sufficient to represent 90% of the AEP. Those N_{sst} combinations represent all the sea states, on which buoy layout optimization is performed.

Finally, a weight $\mathcal{W}_n$ is affected to each of those combinations, to best represent the relative importance of the different sea states in AEP, calculated as follows: $\mathcal{W}_n^{sst} = \frac{AEP(\bar{\theta}, T_p)_n}{\sum_m AEP(\bar{\theta}, T_p)_m}$. Figure 5 shows the 12 $(\bar{\theta}, T_p)$ sea state pairs, with the marker size and colour representing the weight $\mathcal{W}_n^{sst}$ of each pair.

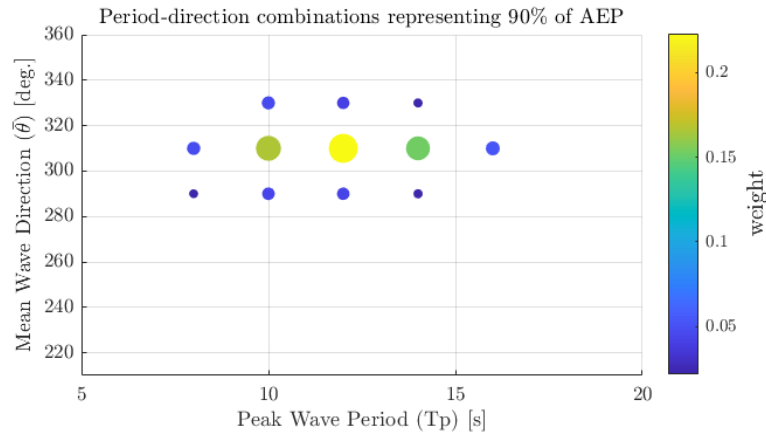


Figure 5: Scatter view of the twelve $(\bar{\theta}, T_p)$ combinations representing 90% of the AEP. Size and colour represent the weight of each pair.

4.3 Accounting for X-band radar complementarity

It is generally considered that X-band radars are able to measure waves in mean wind speeds greater than 3 m/s, and significant wave heights larger than 1.5 m. However, the latter threshold may depend on the radar installation height above the sea surface: A lower height implies larger wave-induced variations of the radar beam incidence angle onto the sea surface, thus better contrast on the radar images, at the expense of a more limited observation range under the effect of wave occlusion. Experiments carried out by IFPEN, not yet published, with an X-band radar mounted at a height of 10 m above the sea surface, showed that significant sea clutter signal could be obtained for significant wave heights as low as 1 m.

For future deployments of the CETO device in a farm, where an X-band radar may be installed on a floating platform, the present buoy layout optimization procedure can be adapted to ensure that the wave buoys will work best, in those sea states where the radar will not be operational. To that end, using wind speed data also available in the WaveScan dataset, a new scatter diagram is built, based only on those WaveScan time steps, where the wave height and wind speed conditions are outside the expected radar operational range. The rest of the procedure remains exactly identical. However, in the prospect of CETO's deployment at the BiMEP test site, where radar availability has not been confirmed, the results presented in the rest of this deliverable do not consider radar complementarity.

5 BUOY MEASUREMENT SYSTEM AND REAL-TIME COMMUNICATIONS

In this section, wave measurement buoy technologies, suitable for the real-time prediction challenge, are identified, and their typical characteristics are described in terms of measured data and accuracy. The buoy mooring arrangements are discussed, since they govern the buoy's "watch circle", i.e. the slow variations in its horizontal position, under the effect of environmental factors, current in particular. This watch circle may have substantial impact on layout optimisation and mean prediction performance. Finally, possible architectures for real-time data transmission to the CETO's onboard computer are outlined, together with their implications in terms of transmission rate and latency.

5.1 Mini wave buoys

If several wave buoys are to be deployed for each device in a WEC farm, those buoys should be relatively inexpensive to buy, deploy, maintain, and replace – otherwise, their benefit will be outweighed by their costs. GPS-based miniature wave boys [17] are good candidates, since their small dimensions and light weight (a few tens of kg) make them easy to deploy by hand from a small embarkation. GPS-based mini-buoys measure vertical and horizontal sea surface displacements, based either directly on GPS position data, or on integration of the GPS doppler velocity data. GPS data is sometimes combined with an onboard IMU.

The most popular commercial mini-buoy is the SofarOcean Spotter buoy. However, as most other wave buoy models, the Spotter buoy does not transmit displacement time series in real-time, but rather, on-board-processed wave spectrum data, at a slow rate suitable for statistical analysis, typically every 30 minutes. In fact, no off-the-shelf commercial wave buoy exists, that transmits sea surface displacement measurements at a rate compatible with the requirements of real-time WEC control. In contrast, the SWIFT buoy, a non-commercial buoy developed by the University of Washington Applied Physics Laboratory, can reportedly [18] transmit its high-frequency (5Hz) motion data in real time to a central receiver, using a radio modem.

The following considerations regarding measurement capabilities, mooring arrangements and real-time communications, are thus strongly inspired by the known characteristics of Spotter and SWIFT buoys.

5.2 Measurement accuracy and sampling rate

Some level of measurement uncertainty needs to be introduced in the wave prediction model, since no wave measurement system can provide perfect sea surface displacement estimates. While wave buoy accuracy is generally assessed through their ability to provide accurate spectral quantities [17], in this work we are interested in the accuracy of their wave-by-wave displacement data. To that end, [19] reports normalised root mean-square (RMS) errors in the order of 5% for horizontal displacements, and 8% for the vertical displacements, in a controlled laboratory environment.

In the remainder of this work, we conservatively model measurement errors as a stationary, Gaussian white noise distribution, with a standard deviation equal to 10% of that of the original signal, for both horizontal and vertical displacements. In practice, this is achieved by adding an appropriately scaled, positive value to the diagonal terms of the covariance matrix Σ_{oo} , defined in Section 2.

Regarding the sampling rate, the Spotter buoy samples the displacements at a 2.5 Hz frequency, while the SWIFT buoy has a higher, 5 Hz sampling frequency. Both are more than sufficient for the 2 Hz requirements stemming from the CETO controller, outlined in Section 3. Therefore, a 2 Hz measurement frequency for the wave motion can safely be assumed throughout the optimization procedure, i.e., with the notations of Section 2, $\Delta t = 0.5$ s.

5.3 Buoy mooring and position variations

In a WEC farm, mini buoys will need to be moored (as opposed to drifting). For accurate wave motion measurements, the mooring system should allow the buoy to move relatively freely. For example, for

the Spotter buoy, Sofar Ocean⁷ recommends a single or double catenary design, illustrated in Figure 6, where the vertical mooring section (which goes from the anchor to the surface float) should be at least 1.2 times the water depth, while the catenary section(s) should be approximately 10 m long.

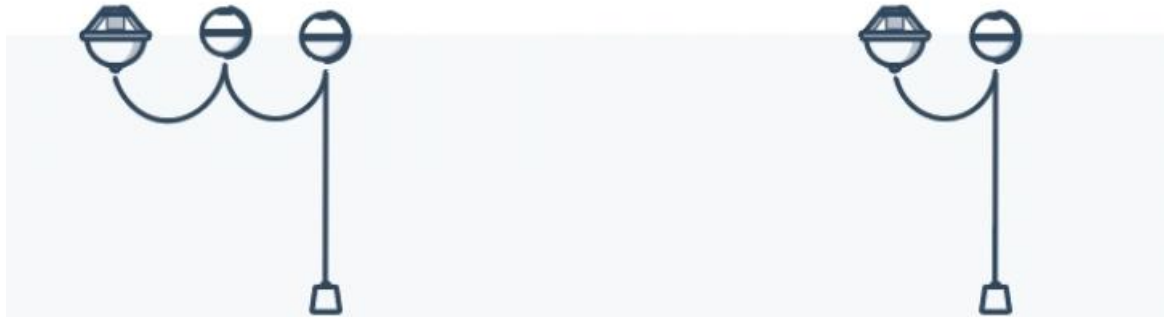


Figure 6: The two recommended Spotter mooring arrangements, reproduced from [Sofar Ocean's website](https://sofarocean.com/spotter-mooring-guidelines)

Such a mooring arrangement results in large horizontal excursions in the wave buoy position along its watch circle. For example, in a 50 m water depth, the upper end of the vertical section can extend up to $50 \times \sqrt{1.2^2 - 1} \approx 50 \times 0.66 \approx 33$ m. Adding one or two catenary lines, horizontal excursions in the order of 45 to 55 m are possible. This is confirmed by considering the GPS position data of a Spotter buoy that was deployed by BiMEP in an approximately 75 m water depth at the up-wave boundary (with respect to the principal wave direction) of CETO's future deployment area (see Figure 3). The 2D histogram of the buoy position, shown in Figure 7, indicates a distribution concentrated around the buoy watch circle, with a mean radius ≈ 40 m.

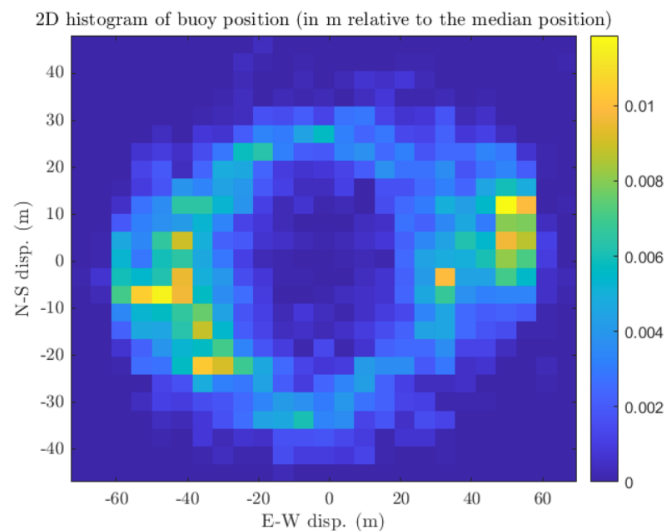


Figure 7: 2D histogram of a Spotter buoy position at the BiMEP test site. The colour indicates the probability of each bin.

A more detailed analysis of the BiMEP dataset highlights that the buoy angular position, along its watch circle, is dictated by current, which changes almost independently from the wave conditions. As a consequence, the buoy position variations throughout the year can be modelled by means of a statistical distribution, independent of the wave condition distribution, and calibrated by means of the

⁷ <https://sofarocean.com/spotter-mooring-guidelines-2-0-3cbda0eccd7c4f44ba1001aa64709bfa>

BiMEP dataset. More precisely, for simplicity, we consider that the buoy position varies along its watch circle, with an angular position distribution calibrated based on the BiMEP available Spotter dataset, as illustrated in Figure 8 below. The watch circle radius is also scaled up to 51 m, to account for the larger 70 m depth at the future CETO deployment berth at BiMEP.

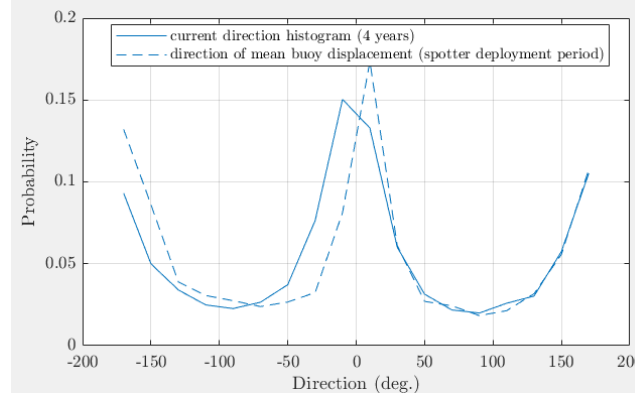


Figure 8: Histogram of the angular position of Spotter buoy deployed at BiMEP (solid line), histogram of the current direction at BiMEP (dashed line)

Furthermore, since the position variation is induced by current, which varies simultaneously for all wave buoys around a CETO device, it can be considered that all wave buoys of the array undergo the same position variation at the same time.

It is extremely important to note that this current-induced variation in the buoy position occurs at a very slow (typically hourly) rate, and that it must clearly be distinguished from the wave-induced orbital buoy displacement, which takes place on a second-by-second basis, and serves as the input to the wave predictor. Therefore, *in a given 30 min or 1 h sea state*, it can be assumed that the buoy's mean position (along its watch circle) is known and constant.

In practice, the buoy layout optimization becomes, strictly speaking, the optimization of the buoy anchoring positions, while the buoys themselves will undergo angular position variations along their watch circle, centred on the anchoring positions. Discretizing the buoy angular position variations into $N_{pos} = 6$ bins, the independence of wave and current conditions implies that, for each calculation of the buoy layout performance in a sea state $(\bar{\theta}, T_p)_n$ of Section 4, the prediction NMSE needs to be calculated for each angular position bin m , with weight $\mathcal{W}_m^{pos}$ equal to the bin probability.

As discussed above, the buoy watch circle radius depends heavily on the mooring configuration. Although Sofar Ocean does not recommend them for their Spotter buoy, other mooring configurations could be considered, allowing a more restricted watch circle. In particular, the SWIFT buoy mooring configuration described in the field experiments of [18] is comprised of a tension mooring, extending from the anchor on the seabed to a subsurface float, 10 m below the sea surface, and a 3:1 scope from the subsurface float to the sea surface. In those experiments, carried out in a 90 m water depth, the achieved watch circle radius is only 15 m. Therefore, such an intermediate scenario will also be considered in the buoy layout optimization.

5.4 Data transmission aspects

As discussed in Section 5.1, no commercial buoy currently exists, which transmits its displacement time series in real time. The buoy technology, closest to the needs of a WEC farm, is indubitably UW-APL's SWIFT v4 buoy [18]. Those are equipped with radio modems (900 MHz) to transmit 5 Hz motion data from the buoy array to a central receiver, with a theoretical range of at least 3 km, and a 1.5 Mbps data rate, more than sufficient to transmit 5 Hz-sampled x, y, z displacement data, GPS coordinates and time stamp. In [18], a 2-months deployment of the buoys is reported, suggesting that real-time communications power requirements are compatible with a relatively long deployment period. It is also claimed that the 5 Hz displacement data is successfully transmitted in real time during the experiment. Furthermore, in the same work, a target of 1 s latency is expressed to provide wave predictions, and it is implied that the main bottleneck towards this goal is in the prediction calculations – rather than the data transmission latency. However, since no analysis of the data transmission latency is provided, it remains unclear whether or not data transmission latency was significant.

In the SM-RTWF methodology, the only computation to carry out in real time is a linear operation onto the observed variables, which can be typically performed in less than a millisecond on a standard laptop. Therefore, apart from the MPC computation time, data transmission becomes the main source of latency.

More precisely, data transmission latency corresponds to the time that elapses, between the instant a wave displacement value is measured or estimated at the buoy, and the instant it is received by the wave predictor, embedded within the CETO's control system. We recommend that data transmission tests at sea should be carried out, to estimate a maximum transmission latency, T_{lat} . Then, it must be considered that, at time t , observed measurements are only available up to time $t - T_{lat}$. In the notations of Section 2, this amounts to setting $\tau_{o,max} = -T_{lat}$. By doing so, the SM-RTWF predictor and buoy layout can both be optimized, taking this data transmission latency constraint into account.

CETO being a submerged point absorber with operating depth varying between 2 m and 30 m, direct buoy-to-WEC communication is expected to be challenging and costly. Therefore, the communication between the predictor's wave buoys and the WEC is assumed to be point-to-point radio between the measurement buoys and the onshore control station, and over fibre-optics from the shore station to the WEC. The additional latency added by the fibre-optics path is expected to be only a few milliseconds, which is negligible in comparison to the 1 s target discussed in [18].

6 OPTIMIZATION METHODOLOGY

We can now start gathering the outcomes of Sections 1-5, to specify the predictor settings and optimization cost function, and define an optimization methodology.

6.1 SM-RTWF variables, implementation and settings

Consider a set of N_b measurement buoys, each of which measuring 3 variables (namely the x, y, z particle displacement at the sea surface, as discussed in Section 5), and with positions $(\vec{x}_1, \dots, \vec{x}_{N_b})$. Thus, in the notations of Section 2, the number of observed variables is at least $M_o = 3N_b$. The predicted variables are excitation forces at the CETO device in 5 DoFs, so $M_p = 5$. Without loss of generality, the CETO is assumed to be at the origin $\vec{x} = (0,0)$.

As mentioned in Section 3.3, we also consider the scenario, where past excitation estimates (carried out by the control system, based on measured WEC dynamics) are included among the observations. In the latter case, the number of observed variables becomes $M_o = 3N_b + 5$.

To fill in the entries of observation and prediction covariance matrices, as outlined in Section 2, the cross-spectra of observed and predicted variables are needed, as per Equations (1) and (2). The calculation of those equations, in turn, uses the directional wave spectrum and the directional transfer functions $H_i(\omega, \theta)$, relating each variable i to the incident free surface elevation.

The transfer functions for the observed wave displacement variables can be obtained from the following three transfer function, stemming from linear wave theory:

- For the z displacement, which is the same as the free surface elevation η , the transfer function is obviously $H_{\eta z}(\omega, \theta) = 1$ everywhere.
- For the x –wise displacement, $H_{\eta x}(\omega, \theta) = \frac{j \cos \theta}{\tanh kh}$
- For the y –wise displacement, $H_{\eta y}(\omega, \theta) = \frac{j \sin \theta}{\tanh kh}$

The transfer function for the 5 excitation force variables are those, given by NEMOH⁸, a boundary element method (BEM) hydrodynamic solver.

The time step Δt of Section 2 is set to 0.5 s, a value that is suitable for the control requirements highlighted in Section 3.4, and compatible with the sampling and data transmission rate capabilities of typical GPS mini buoys, detailed in Section 5. The minimum observation time $\tau_{o,min}$ is set to -100 s, while the maximum observation time $\tau_{o,max}$ is set to $-T_{lat}$ (for buoy measurement variables, subject to data transmission latency, as explained in Section 5.3) or 0 (for excitation estimates, when included among the observations). The minimum prediction time is $\tau_{p,min} = 0$ s, and the maximum prediction time is $\tau_{p,max} = T_h^*$ (see Section 3).

6.2 Cost function evaluation

Consider a number N_b of buoys, with anchor positions in a set of positions $X = (\vec{x}_{a,1}, \dots, \vec{x}_{a,N_b})$. X is the optimization variable, and we want to define and calculate the cost function resulting from choosing a particular value of X . We start by calculating the cost function in a given sea state $S_n = (\bar{\theta}, T_p)_n$, and under a certain buoy position variation $\vec{\delta}_p$ (see Section 5.3). Thus, the $3N_b$ buoy-measured variables have positions $X_b = (\vec{x}_{a,1} + \vec{\delta}_p, \dots, \vec{x}_{a,N_b} + \vec{\delta}_p)$.

The spectrum $S(\omega, \theta)$ is calculated from $(\bar{\theta}, T_p)_n$ using the JONSWAP-Mitsuyasu formulation described in Section 4, and assuming $H_s = 1$ m (without loss of generality, since NMSE results are independent on the scale). Thus,

- the positions in X_b ,
- the time settings described in Section 6.1,
- the transfer functions described in Section 6.1,

⁸ <https://gitlab.com/lheea/Nemoh>

- the spectrum $S(\omega, \theta)$,

are all employed in Equation (1), to fill in the entries of covariance matrices Σ_{oo} , Σ_{op} and Σ_{pp} . Those allow the calculation of the optimal prediction NMSE for each DoF $j = 1 \dots 5$ and each time horizon $\tau_k = 0, \dots, T_h^*$, $NMSE_j(\tau_k)$. Those 5 NMSE profiles must all be combined into a single scalar value, which is achieved using the horizon weighting function $\mathcal{W}^{hor}(\tau)$ and DoF weights $\mathcal{W}_j^{DoF}$, defined in Section 3, to obtain an NMSE index, for sea state n and buoy position p :

$$I_{NMSE}(n, p) = \sum_j \mathcal{W}_j^{DoF} \sum_k \mathcal{W}^{hor}(\tau_k) NMSE_j(\tau_k) \quad (7)$$

There remains to combine $I_{NMSE}(n, p)$ for all sea states and buoy position variations, which is again achieved using the appropriate weights defined in Sections 4 and 5:

$$CF = \sum_n \mathcal{W}_n^{sst} \sum_p \mathcal{W}_p^{pos} I_{NMSE}(n, p) \quad (8)$$

which is the cost function we seek to minimize.

6.3 Optimization algorithm

It was decided to optimize buoy placement iteratively, i.e. starting with one buoy, then adding a second one, etc. This has the benefit of computational simplicity; additionally, the results become easy to interpretate and introduce a clear hierarchy among buoys: Each new buoy brings less benefit, in terms of cost function, than the previous one.

Therefore, one ends up with a sequence of single-buoy placement problems. Although an SQP algorithm was successfully tested to solve those single-buoy problems, an exhaustive search approach, whereby the buoy position is optimized by testing all positions in a discretized grid, was found even more advantageous:

- Since the space of possible buoy positions is explored, the outcome of each optimization is a cost function map, which provides interesting insight on the buoy placement;
- The main computational bottleneck, in the cost function evaluation, is the update of the covariance matrices. Let N be the number of existing buoy, and we are placing buoy $N + 1$. In an SQP algorithm, at a new cost function evaluation, a number of entries proportional to $(N + 1)^2$ will need to be updated, for each spectrum n and each buoy position variation p . So, if there are on average N_{cfe} cost function evaluations in the SQP solution, N_{sst} sea states and N_{pos} position variations, the computation time for cost function optimization is proportional to $N_{cfe} N_{sst} N_{pos} (N + 1)^2$. In the exhaustive search approach, we can first fix the sea state and buoy position, and make the new buoy position explore the whole search space, before moving on to the next sea state and buoy position. When doing so, only the indices of the covariance matrices, that relate to the new buoy, need to be updated, and the number of such indices is proportional to $N + 1$. If there are N_{grid} possible placements in the exhaustive search space for the new buoy, the computation time for the exhaustive search optimization becomes proportional to $N_{grid} N_{sst} N_{pos} (N + 1)$. Therefore, the computation time increases linearly, instead of quadratically, with the number of buoys.

7 RESULTS

7.1 Verification of the optimization process

We start by illustrating the optimization process with first sample results, shown in Figure 9. In this example, the buoy position variations are neglected (i.e. the buoy watch circle radius is set to zero). Furthermore, the parameter α is set to zero (no preference for present), and excitation estimates are not included among the observations.

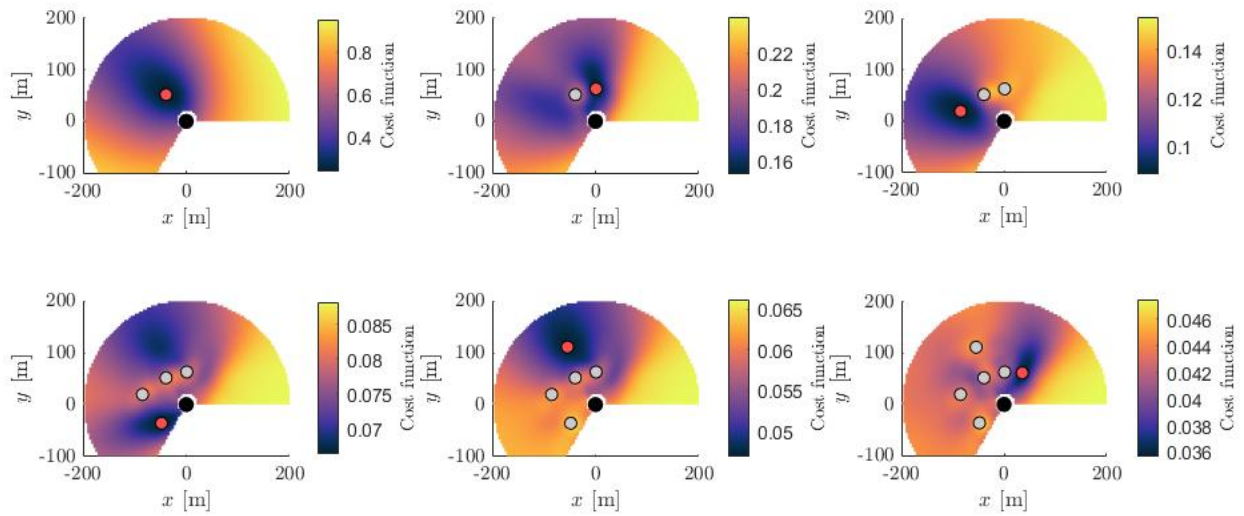


Figure 9: Sequential optimization of 6 wave buoy locations. Each subplot shows the cost function map for the sub-problem. Grey dots: locations previously optimized buoys, and red dot: optimal location of the new buoy.

Each subplot of Figure 9 shows the cost function map, for the sub-problem corresponding to the addition of a new wave buoy. The grey dots represent the locations of the existing, already optimized buoys, and the red dot represents the optimal location of the new buoy, added in each sub-problem. Unsurprisingly, the first buoy is placed up-wave, relative to the prevailing wave directions at BiMEP (310° relative to north, see Figure 4). Then, the 2nd and 3rd buoys are placed, so as to cover a broader set of directions. The 4th and 6th buoy positions allow the anticipation of lateral wave components. The 5th one is placed further up-wave, suggesting that it improves the NMSE over the larger time horizons.

We can also note that, in each new sub-problem, local maxima of the cost function appear around existing buoys: This is because placing a new buoy close an existing one will barely bring any novel statistical information. Thus, the algorithm naturally places the buoys at some distance from each other.

7.2 Preliminary optimization scenarios

Four baseline optimization scenarios are first considered, where the buoy mooring system is assumed ideal (i.e. the watch circle radius is 0) and data transmission latency time is assumed to be 0. Those four scenarios correspond to each possible combination of the following parameter choices:

- Depending on whether or not past excitation estimates are included among observations;
- Depending on whether the horizon-weighting parameter α is set to 0 (no preference for present) or 1 (marked preference for present).

The layout optimization is performed up to 6 buoys in each of those 4 scenarios. Subsequently, numerical simulations of the predictor are run for each of the optimization sea states, thus providing simulated time series of CETO excitation forces, along with the predictions achieved at each time step by the optimized predictor. Those simulation results are used by CTS in their MPC simulation framework, to evaluate the AEP, obtained in each scenario.

Thus, those 4 scenarios allow us to assess the impact (in terms of buoy layout strategy, cost function and AEP) of:

- including excitation among observed variables;
- horizon-weighting the NMSE performance criterion.

The cost function value should be thought of as a weighted average of the annual mean prediction NMSE, over the prediction time horizon – with larger weight given to short time horizons, when α is set to 1. Figure 10 shows the cost function decrease with the number of buoys, without modelling the buoy watch circles. In all cases, including excitation estimates among the observations allows a substantial decrease in the cost function: As a result, significantly fewer buoys are required to achieve a prediction performance similar to the case where only wave buoy measurements are observed. This benefit is even stronger when preference is given to short time horizon (right hand-side graph): Indeed, past excitation estimates provide very accurate excitation predictions for the shortest time horizons. In all cases, 6 buoys allow a cost function well below 5%, and as low as 1.5% when excitation estimates are included and α is set to 1.

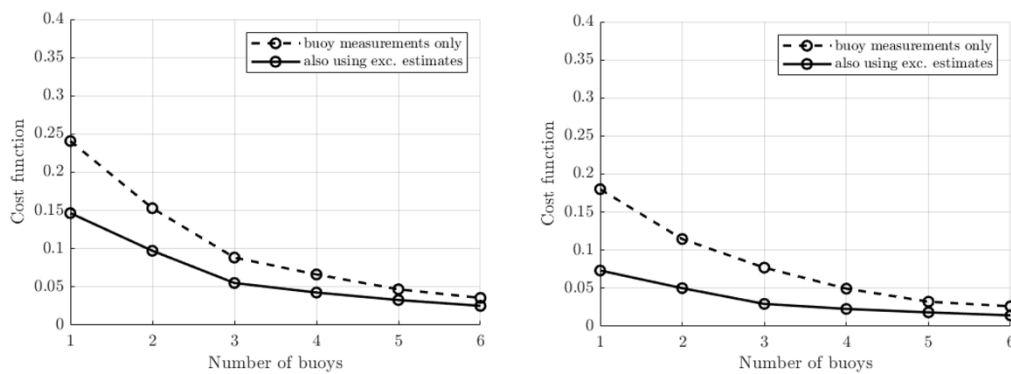


Figure 10: Cost function vs number of buoys, without buoy position variations (idealised moorings). Dashed lines: Only buoys measurements are included in the observations. Solid lines: Past excitation estimates are also included. Left: $\alpha = 0$ (no preference for present), right: $\alpha = 1$ (preference for present)

It is now examined, in Figure 11, how those 4 scenarios influence the optimal choice of buoy layout. All layouts are approximately contained within a 100 m radius from the CETO. When excitation estimates are not included among the observations (top graphs), introducing preference for present requires wave buoys, significantly closer to the CETO (top right graph): Indeed, short up-wave distances are appropriate for short horizons. In contrast, when excitation is included among observations, the

optimal layout is relatively consistent, whether or not preference for present is introduced. Furthermore, wave buoys tend to be placed at a larger distance in that case. An interpretation is that excitation estimates allow accurate predictions in the very short term, while wave buoys take the role of improving the prediction over the longer horizons.

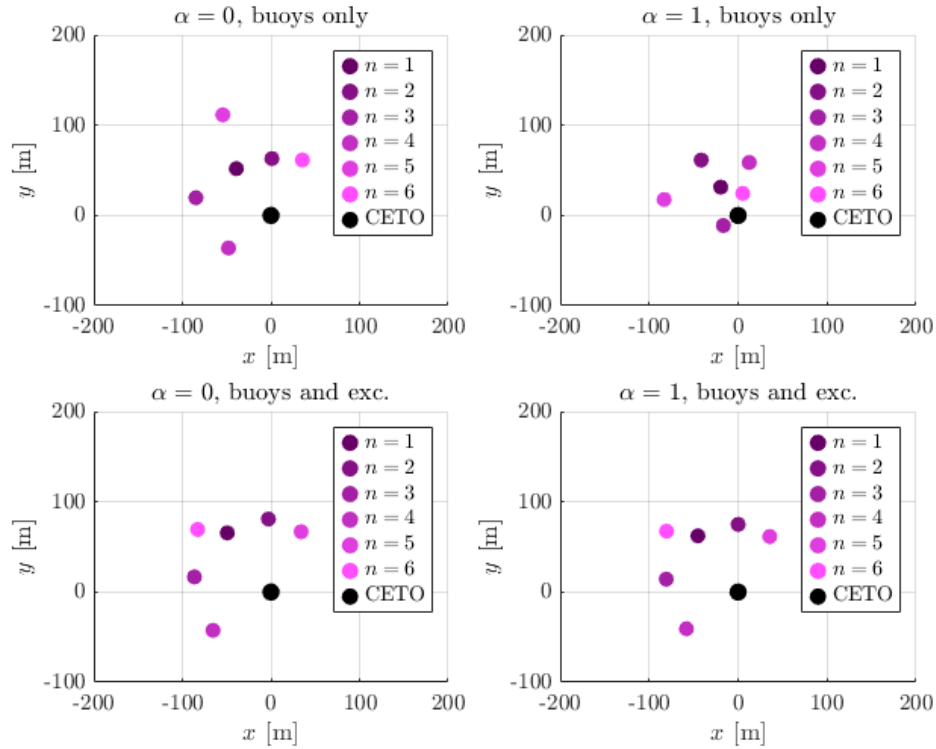


Figure 11: Optimized layout for the six buoys, without buoy position variations (idealised moorings). Top: Only buoys measurements are included in the observations. Bottom: Past excitation estimates are also included. Left: $\alpha = 0$ (no preference for present), right: $\alpha = 1$ (preference for present).

Finally, and most importantly, simulated time series of the 4 baseline scenarios are provided as inputs to CTS's MPC simulator for AEP assessment. Those AEP values are normalised by the AEP obtained under ideal predictions, and the results are shown in Figure 12. The most striking outcome is that all scenarios perform extremely well, in the sense that, even with one buoy only, the relative AEP loss is less than 2%, and becomes negligible beyond 4 buoys. In those scenarios where excitation estimates are included among observed variables, relative AEP losses are the smallest, and near-ideal power capture is achieved with as little as 3 buoys. In those cases, introducing the horizon weighting parameter does not have any impact, because the shortest time horizons (favoured with $\alpha = 1$) are well predicted using past excitation forces, regardless of the buoy layout. Without excitation estimates, the AEP loss is slightly more substantial, but near-optimal power capture is still achieved using 3 or 4 buoys. In those cases, introducing preference for present in the optimization ($\alpha = 1$) allows a more rapid drop in AEP losses.

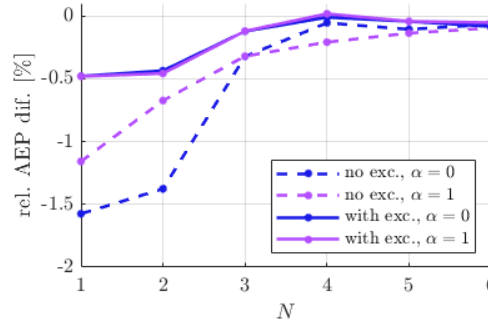


Figure 12: Relative AEP in the 4 baseline scenarios. Dashed lines: Only buoy measurements are included in the observations. Solid lines: Past excitation estimates are also included. Light blue: $\alpha = 0$ (no preference for present), dark blue: $\alpha = 1$ (preference for present)

Despite the clear benefit of including excitation force estimates among observations, the decision to actually include them deserves discussion. Indeed, excitation estimates depend on how the WEC dynamics are excited by waves and the control system. In turn, the control system behaviour depends on the excitation predictions. If those predictions are dependent on excitation estimates, it should be ascertained that the overall estimation-prediction-control system is not affected by undesirable closed-loop behaviour, which was not possible to test at this stage. Therefore, it was decided to move forward with the more prudent approach of optimizing the wave buoy layout, assuming only buoy measurements are available. When the closed loop estimation-prediction-control can be characterized in closed-loop, either experimentally or in numerical simulations, excitation estimates can still be included at a later stage, while keeping the same buoy layout (which will still improve the prediction quality).

Furthermore, because introducing preference for present brings substantial benefits when only buoy measurements are used, setting $\alpha = 1$ is a consistent choice.

In short, before introducing more realism into the optimizations by considering buoy position variations and data transmission latency, it was decided to stick to the optimization scenario where only wave buoy measurements are used and α is set to 1.

7.3 Buoy position variations

Now modelling the buoy mooring watch-circle, the search space is adjusted so that the watch circle cannot enter in collision with the CETO. Also bear in mind that, as mentioned in Section 5.3, the buoy position variations, along their watch circle, is dictated by current, so that all buoys are in the same angular position simultaneously, and buoy-to-buoy collisions are impossible. Two mooring circle radii are considered: a 51m, “Spotter-like” watch circle radius, and a 15m, “SWIFT-like” watch circle radius.

Figure 13 shows the optimization cost function in those two mooring scenarios, together with the ideal mooring scenario. Unsurprisingly, the existence of buoy position variations has an adverse effect on the cost function, all the more so that the watch circle radius is larger. For the larger 51m-radius watch circle, the optimized cost function values almost double, with respect to the idealized mooring scenario. In contrast, for the smaller, 15m-radius watch circle, the optimized cost function is barely affected.

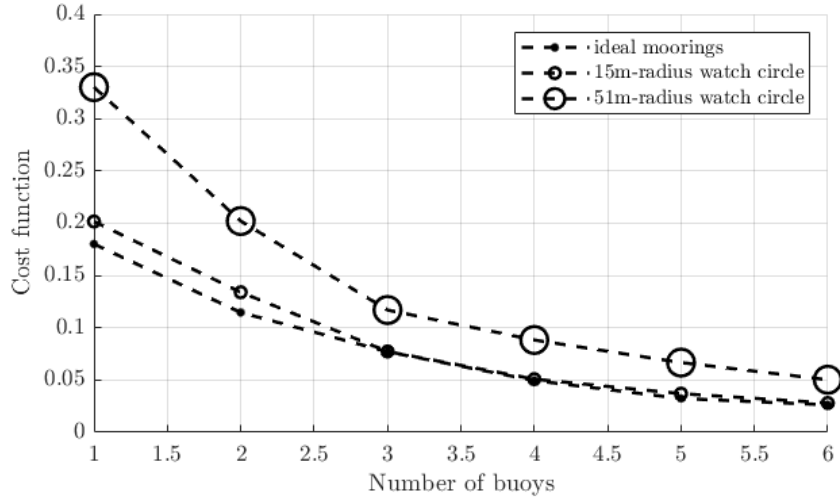


Figure 13: Cost function vs number of buoys, in different buoy mooring modelling scenarios. In all cases, only buoy measurements are included in the observations, and $\alpha = 1$ (preference for present).

The corresponding buoy layouts are compared in Figure 14. In each subfigure, the coloured dots represent the buoy mooring locations, while the buoy watch circles are represented in grey, dashed circles (in the rightmost two subfigures). Overall, except for the first buoy, of which the optimized position remains relatively consistent across cases, the optimal buoy layout appears to depend strongly on the buoy watch circle modelling. Besides, with a larger watch circle, buoys tend to be located further away from the CETO. Even with the small 15m-radius watch circle, the difference in buoy layout is considerable, with respect to the idealized-mooring case – although the resulting optimal cost function values are almost identical, as shown in Figure 13. This demonstrates the importance of appropriately modelling the buoy mooring watch circle when optimizing the buoy layout.

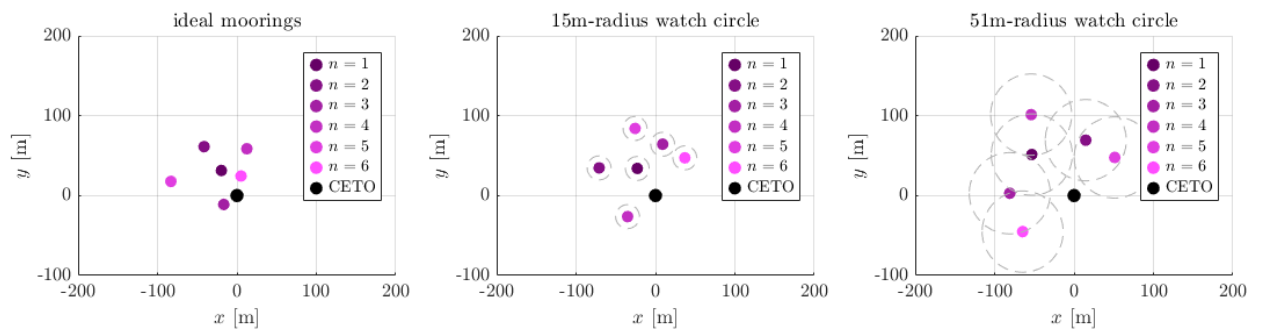


Figure 14: Optimized buoy layout for the six buoys, in different buoy mooring modelling scenarios. The buoy watch circles are represented by grey, dashed circles. In all cases, only buoy measurements are included in the observations, and $\alpha = 1$ (preference for present).

Finally, the impact of the buoy watch circle in terms of AEP is examined in Figure 15. With the larger, 51m-radius watch circle, the impact is found to be very significant: With $N = 1$ buoy, the AEP drops by nearly 5% with respect to the “ideal prediction” AEP (while the drop is only 1.2% with the idealized

mooring). However, beyond $N = 3$ buoys, very satisfactory results are recovered, with AEP losses below 0.5%. With the smaller, 15m-radius watch circle, the AEP impact is within the simulation statistical variability, consistent with the cost function being almost identical to the ideal mooring case (Figure 13).

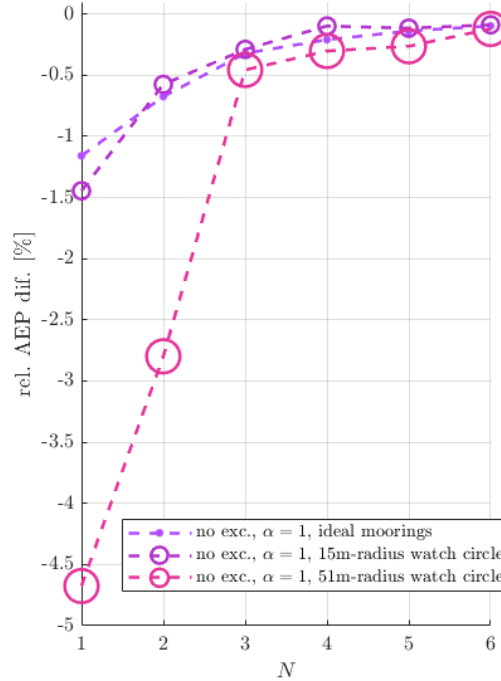


Figure 15: Relative AEP in the 3 mooring scenarios, when only buoy measurements are included in the observations, and $\alpha=1$ (preference for present).

From the above comparison, it appears that a large buoy watch circle worsens the average prediction quality and, hence, AEP, but that the impact is almost insignificant for a number of buoys beyond $N = 3$. The intermediate case of a “SWIFT-like”, 15m-radius watch circle is retained for the rest of this study, because this mooring arrangement is realistic and provides AEP results, identical to the “idealized mooring” scenario.

7.4 Consideration of data transmission latency

In this last subsection, data transmission latency is considered in the optimization, and its impact on the results are examined. As discussed in Section 5, it is too early to know precisely the data transmission performance which will be achieved with the planned communication system among the CETO and its associated wave buoys, although 1s seems to be a likely order of magnitude. In the following, a conservative 5s latency time is considered, to demonstrate that the data transmission latency can be duly considered in the present buoy layout optimization method, and to obtain preliminary estimates of the results sensitivity to this modelling parameter.

Figure 16 shows the optimized cost function value as a function of the number of buoys, in the two cases $T_{lat} = 0s$ and $T_{lat} = 5s$. As may be expected, the data latency time has an adverse effect onto the achievable cost function value.

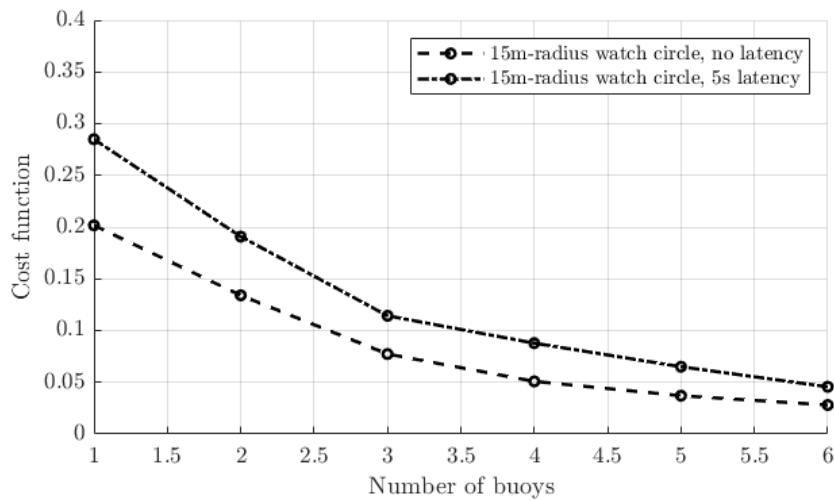


Figure 16: Cost function vs number of buoys, depending on whether data transmission latency is 0s (dashed lines) or 5s (dash-dot). In both cases, a 15m-radius watch circle is considered, only buoy measurements are included in the observations, and $\alpha = 1$ (preference for present).

Figure 17 visualises how the latency time influences the optimal buoy layout. The impact of latency time can now be interpreted more clearly: To compensate for the added delay between the measurement instant and the data reception instant, buoys need to be placed further up-wave. This greater distance makes it more difficult for the buoys to capture multi-directionality.

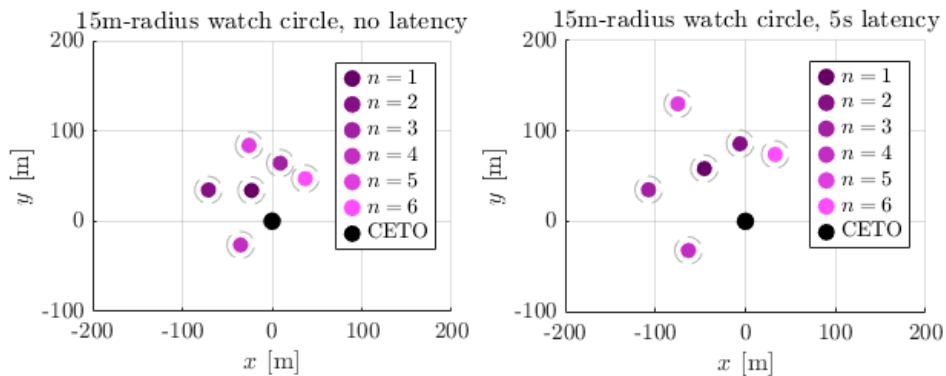


Figure 17: Optimized buoy layout for the six buoys, depending on whether data transmission latency is 0s (left) or 5s (right). In both cases, a 15m-radius watch circle is considered, only buoy measurements are included in the observations, and $\alpha = 1$ (preference for present).

Finally, Figure 18 shows that the latency time impact, in terms of AEP, is substantial. Nevertheless, with 5 buoys, the AEP becomes similar to the “perfect prediction” case.

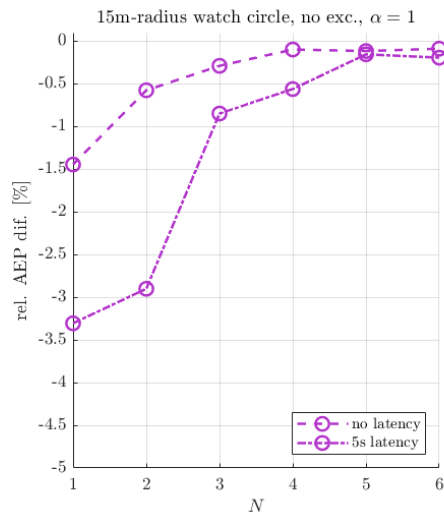


Figure 18: Relative AEP with a 0s (dashed lines) and a 5s (dash-dot) data transmission latency time, in the 15m-radius watch circle mooring scenario, when only buoy measurements are included in the observations, and $\alpha=1$ (preference for present).

8 CONCLUSION

A buoy layout optimization methodology has been developed, with the SM-RTWF framework as a fundamental building block (Section 2), to maximize the accuracy of real-time excitation predictions onto the CETO device. A maximum time horizon of 15 s stems from the CETO MPC requirements. A 0.5 s prediction time step is found appropriate, also equal to the prediction update time. The wave buoys measure the vertical and horizontal displacements of the sea surface. Additionally, (past) excitation force estimates, calculated in the MPC control system, can also be included among the input variables of the predictor.

While the SM-RTWF framework provides a horizon-dependent NMSE profile, for each hydrodynamic degree of freedom, and in a given sea state, there remains to build a single-valued cost function, by combining those NMSE values for all time horizons, hydrodynamic DoFs, and environmental conditions representative of the BiMEP site. The determination of appropriate weights to achieve this combination is addressed in Sections 3 to 5.

As described in Section 6, the cost function calculation is then embedded within a relatively simple optimization algorithm, whereby the wave buoys are added iteratively, and each new buoy is optimally placed using an exhaustive search approach. For each optimized configuration, numerical simulations are run and provided as inputs to a simulator of the CETO MPC system, to estimate the resulting AEP.

A number of modelling scenarios and optimization options are investigated in Section 7. In particular, the benefit of including (or not) excitation estimates among observed variables, and that of weighting (or not) the prediction horizon to give preference to short time horizons, are assessed. The sensitivity of the optimal layout and resulting performance to the buoy mooring watch circle is also investigated, as well as the impact of data transmission latency time.

Despite the simplicity of the optimization algorithm, and depending on the modelling scenarios considered, between 1 and 5 optimally placed buoys are sufficient to allow AEP losses below 0.5%, with respect to that, obtained with perfect predictions. However, results also show that the optimal layout strongly depends on the modelling scenarios, in terms of mooring assumptions, latency time, inclusion of excitation estimates as observations, etc. This stresses the importance of considering those modelling parameters, when optimizing the buoy layout.

In terms of compromise between feasibility and performance, the retained reference scenario is that of a wave predictor using wave measurements only (i.e. not including excitation estimates among the observed variables), and a mooring arrangement allowing a 15m-radius buoy watch circle. In such a case, no appreciable AEP improvement is observed beyond $N = 4$ buoys. Therefore, moving forward towards the next steps of the COIN project in Work Packages 4 and 5, the reference buoy layout corresponds to the first 4 buoys in the middle graph of Figure 14.

Data transmission latency time can be successfully considered in the optimization. Assuming a (likely exaggerated) 5s latency time, the buoys must be placed significantly further away from the CETO, and 5 buoys, instead of 4, are needed to achieve similar performance. However, it is too soon to retain a particular value for the latency time, in advance of a field assessment of the future real-time communications architecture. For the next steps of the COIN project, involving offline training and validation in numerical simulations and wave basin experiments, latency can be safely neglected.

Finally, if it is determined that excitation estimates can be included among the observations, without inducing unwanted closed-loop instability in the overall estimation-prediction-control real-time process, the results are expected to be even better, which may further reduce the necessary number of wave buoys.

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